RO-003-001543 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

February - 2019

Statistics : S - 502

(Mathematical Statistics) (Old Course)

Faculty Code : 003
Subject Code : 001543

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Question one carries 20 mars, Question - 2 and Question-3 carry 25 marks.
(3) Student can use their own scientific calculator.

1 Filling the blanks and short questions : (Each 1 mark) **20**

1. __________ is a characteristic function of Normal distribution.
2. __________ is a characteristic function of Binomial distribution.
3. __________ is a characteristic function of Chi-square distribution.
4. Measured of Kurtosis coefficient for Normal distribution are __________.
5. If x follows Gama distribution with parameter p then $\mu_4 = k_4 + 3k_2^2$ is __________.
6. If two independent variates $X_1 \sim \gamma(n_1)$ and $X_2 \sim \gamma(n_2)$ then $X_1 + X_2$ is distributed as __________.
7. A linear combination of independent normal variates is also __________.
8. For Normal distribution $\mu_4 = k_4 + 3k_2^2$ is __________.

9. If χ_1^2 and χ_2^2 are two independent Chi-square variates with d.f. n_1 and n_2, respectively, then the distribution of $\frac{\chi_1^2/n_1}{\chi_2^2/n_2}$ is ____________.

10. If two independent variates $X_1 \sim \gamma(n_1)$ and $X_2 \sim \gamma(n_2)$ then $\frac{X_1}{X_1+X_2}$ is distributed as ____________.

11. t–distribution with 1 d.f. reduces to ____________.

12. Weibull distribution has application in ____________.

13. If $X \sim N(0,1)$ and $Y \sim \chi_n^2$, the statistic $\frac{\sqrt{n}X}{\sqrt{Y}}$ is distributed as ____________.

14. A measure of linear association of a variable say, X_1 with a number of other variables X_2, X_3, X_4,............,X_k is known as____________.

15 The range of partial regression coefficient is____________.

16. Define Bivariate Normal distribution.

17. Define Beta second kind distribution.

18. Write mean and variance of Gama distribution with parameter (α, p)

19. Write mean and variance of Normal distribution.

20. Write mean and variance of Gama with parameter p distribution.

2 (a) Write the answer any THREE (Each 2 marks) **6**

1. Define Convergence in Probability.
2. Why characteristic function need?
3. Define Beta first kind distribution.
4. Prove that, if $r_{12} = r_{13} = r_{23} = \rho$ then $r_{31.2} = \frac{\rho}{1+\rho}$

5. Prove that $b_{12.3} = \frac{b_{12} - b_{13}\, b_{23}}{1 - b_{13}\, b_{23}}$

6. In trivariate distribution it is found that $r_{12} = 0.77$, $r_{13} = 0.72$ and $r_{23} = 0.52$.
Find (i) $r_{12.3}$ (ii) $R_{1.23}$

(b) Write the answer any THREE (Each 3 marks) **9**

1. Usual notation of multiple correlation and multiple regression, prove that

$$\sigma_{1.23}^2 = \sigma_1^2 \left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)$$

2. Obtain Probability density function for the characteristic function

$$\phi_{X(t)} = e^{-\left(\frac{1}{2}t^2\sigma^2\right)}$$

3. Define Exponential distribution and obtain its MGF. From MGF obtain its mean and variance.
4. Define truncated Binomial distribution and also obtain its mean and variance.
5. Usual notation of multiple correlation and multiple regression, prove that

$$b_{12} = \frac{b_{12.3} + b_{13.2}b_{32.1}}{1 - b_{13.2}b_{31.2}}$$

6. Prove that $\mu_r = (-i)^r \left[\frac{d^r}{dt^r}\phi_u(t)\right]_{t=0}$; where $u = x - \mu$

(c) Write the answer any TWO : (Each 5 marks) **10**

1. Obtain marginal distribution of x for Bi-variate distribution.
2. Derive t-distribution.

3 If x and y are independent χ^2 variates with n_1 and n_2 degree of freedom respectively then obtain distribution of $\frac{x}{x+y}$ and $x+y$.

4. State and prove that Chebchev's inequality.

5. Usual notation of multiple correlation and multiple regression, prove that

$$R^2_{1.23} = \frac{r^2_{12} + r^2_{13} - 2r_{12}r_{23}r_{13}}{1 - r^2_{23}}$$

3 (a) Write the answer any THREE : (Each 2 marks) **6**

1. Obtain characteristic function of Poisson distribution with parameter λ.
2. Define Log Normal distribution when $y = \log_e(x-a)$
3. Obtain relation between t and F distribution.
4. Prove that $\theta_X(0) = 1$ and $|\phi_x(t)| \le 1$
5. Prove that

$$\sigma^2_{3.12} = \frac{\sigma^2_3\left(1 - r^2_{12} - r^2_{23} - r^2_{13} + 2r_{12}r_{23}r_{13}\right)}{\left(1 - r^2_{12}\right)}$$

6. In trivariate distribution it is found that $\sigma_1 = 2, \sigma_2 = \sigma_3 = 3,\ r_{12} = 0.7, r_{23} = r_{31} = 0.5$, find : (i) $b_{13.2}$ (ii) $\sigma_{3.12}$

(b) Write the answer any THREE : (Each 3 marks) **9**

1. Define truncated Poisson distribution and also obtain its mean and variance.
2. Obtain Harmonic mean of $X \sim \gamma(\alpha, p)$.
3. Obtain mean and variance of Uniform Distribution.

4. Prove that $\mu_r' = (-i)^r \left[\frac{d^r}{dt^r} \phi_X(t) \right]_{t=0}$

5. Usual notation of multiple correlation and multiple regression, prove that

$$r_{xy} + r_{yz} + r_{xz} \geq -\frac{3}{2}$$

6. Usual notation of multiple correlation and multiple regression, prove that

$$b_{12.3}\, b_{23.1}\, b_{31.2} = r_{12.3}\, r_{23.1}\, r_{31.2}$$

(c) Write the answer any TWO : (Each 5 marks) **10**

1. Derive Normal distribution.
2. Obtain relation between F and χ^2.
3. Obtain MGF of Gamma distribution with parameters α and p . Also show that $3\beta_1 - 2\beta_2 + 6 = 0$
4. If the joint pdf of x and y is

$$f(x, y) = \frac{1}{2\pi} e^{-\frac{1}{2(1-\rho^2)}\left\{x^2 - 2\rho xy + y^2\right\}}$$

where $-\infty \leq x, y \leq \infty; -1 \leq \rho \leq 1$

then find (i) Marginal distribution of y. (ii) Conditional distribution of x when y is given.

5. Usual notation of multiple correlation and multiple regression, prove that

$$r_{12.3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{\left(1 - r_{13}^2\right)\left(1 - r_{23}^2\right)}}$$
